

Domain-area distribution anomaly in segregating multi-component superfluids

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The domain-area distribution in the phase transition dynamics of Z_2 symmetry breaking is studied for quasi-two-dimensional multi-component superfluids. The distribution is divided into microscopic and macroscopic regimes with distinct power-law exponents. The macroscopic regime universally exhibits Fischer's law in percolation theory, while the microscopic regime depends on the microscopic dynamics of the system. Numerical experiments of segregating superfluids reveal that the exponent in the microscopic regime reaches its theoretical upper limit in the presence of the circular vortex sheet described by quantum hydrodynamics. An analogy to quantum turbulence and the applicability to different multi-component superfluids are also discussed.

In the phase transition dynamics of spontaneous symmetry breaking (SSB), a number of topological defects are nucleated, forming a complicated network or texture in order parameter fields. This type of phenomenon can occur universally in systems ranging from condensed matter to cosmology and high energy physics [1–4]. The dynamic-scaling law in phase ordering kinetics hypothesizes that the growth of order parameter fields preserves the statistical similarity of the spatial patterns [5]. This hypothesis has been accepted empirically by observing that the structure factors or correlation functions of the fields fit a universal function after rescaling length by the characteristic size l , which obeys a power law $l \propto t^{1/z}$ with the dynamic exponent z .

The application of percolation theory [6] to SSB development has recently attracted attention [7–9] and has become an important problem in statistical physics [10]. Since percolation theory reveals different statistical aspects of the phase ordering kinetics, such studies could lead to a greater understanding of the highly non-equilibrium physics of SSB development. For example, the most fundamental problem of phase ordering kinetics [5] is two-dimensional Z_2 symmetry breaking in which the order parameter is a real scalar field and the defect is a linear object called a domain wall. One significant prediction of percolation theory is that the time development of the number distribution $\rho(S, t)$ of domains of area S obeys a universal power law $\rho \propto S^{-\tau_F}$ with the Fisher exponent $\tau_F = 187/91 \approx 2$ of two-dimensional percolation [6].

There has been growing interest in the phase ordering kinetics of atomic Bose-Einstein condensates (BECs), and the dynamic-scaling law in superfluid systems has been investigated theoretically in different situations [11–18]. Recently, percolation theory has been applied to the segregation of binary BECs in quasi-two-dimensions, and the dynamic finite-size-scaling analysis revealed that the domain structures preserve the percolation criticality during Z_2 symmetry breaking with the percolation threshold $p_c = 0.5$ [19]. Recently, the domain-area distribution in segregating superfluids was shown to obey Fisher's power law for large-scale structures [20, 21]. In-

terestingly, the distribution of domains smaller than the characteristic domain size l in the superfluid systems shows anomalous behavior different from that observed in conventional coarsening systems of nonconserved and conserved fields [7, 22]. These results suggest that the dynamic-scaling hypothesis holds even when the quantum effect becomes important in superfluid systems since the small-scale structure strongly depends on the “microscopic” dynamics of the domain wall in the system under consideration. This anomalous behavior of smaller domains indicates the existence of a different dynamic-scaling regime that reflects the quantum hydrodynamics, namely, “microscopic” dynamics in quantum fluids. Can such a scaling regime firmly coexist with the universal scaling regime of percolation theory in superfluid coarsening systems? If so, the critical exponent and the effect of the microscopic nature on the scaling behavior should be important for deeper understanding of the physics of SSB development.

To answer these questions, we demonstrate the dynamic-scaling analysis according to the combined theory of phase ordering kinetics and percolation, called *phase ordering percolation*. The theoretical conjecture presented here is summarized in Fig. 1. Assuming an asymptotic form of the domain-area distribution, this theoretical analysis is universally applicable to different coarsening systems. The domain-area distribution has two distinct scaling regimes, namely the microscopic and macroscopic regimes, which are characterized by the exponents τ_{mic} and $\tau_{\text{mac}} = \tau_F$, respectively. Numerical experiments of binary BECs reveal that the microscopic exponent τ_{mic} of this system takes the theoretical upper limit $\tau_{\text{mic}} = 3/2$, which is quite different from those of conventional coarsening systems. This anomalous behavior is connected to the quantum-fluid effect of a circular vortex sheet by finding a characteristic scaling law derived from the quantum hydrodynamics. Finally, an analogy to hierarchy in quantum turbulence and the applicability of this scaling theory to different multi-component superfluids will be discussed.

We first formulate a generalized model for evaluating the scaling behavior of phase ordering percolation before

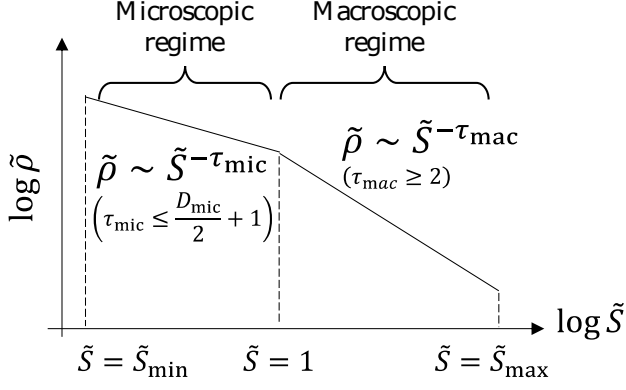


FIG. 1: The expected asymptotic form of the rescaled domain-area distribution $\tilde{\rho}$.

discussing superfluid systems. Consider the time evolution of domain-area distribution ρ in a coarsening system that undergoes Z_2 symmetry breaking in two dimensions. There are two kinds of domains, namely \uparrow - and \downarrow -domains. We write the number of \uparrow - or \downarrow -domains, which have areas between S and $S + dS$, divided by the system area L^2 at time t as $\rho(S, t)dS$. According to the theory of phase ordering percolation, $\rho(S, t)$ is described by the non-dimensional universal function $\tilde{\rho}(\tilde{S})$ of $\tilde{S} = S/S_l$ with characteristic domain area $S_l = \pi l^2$ and mean domain size l :

$$\tilde{\rho}(\tilde{S}) = S_l^2 \rho(S, t). \quad (1)$$

This is another expression of the dynamic-scaling hypothesis in the sense that the hypothesis is conventionally described by the structure factor or the correlation function [5]. The dynamic-scaling law (1) has been experimentally [8] and numerically [7, 20–22] confirmed in different systems.

To analytically evaluate the rescaled distribution function (1), it is convenient to introduce the normalization conditions for the domain-area distribution. The condition or sum rule for the total domain area was demonstrated in Ref. [7]. Here, we formulate the condition for the total domain-wall length $R(t) = L^2/l(t)$ that yields useful input for the analysis of microscopic exponent τ_{mic} .

Because domain walls exist between \uparrow - and \downarrow -domains, the length R is calculated by integrating the total length of the walls surrounding all \uparrow - or \downarrow -domains. Thus, by using the length $l_w(S)$ of the wall surrounding a domain of area S , we have the normalization condition

$$\frac{R}{L^2} = \int_{S_{\text{min}}}^{S_{\text{max}}} l_w \rho dS = \frac{1}{l} \int_{\tilde{S}_{\text{min}}}^{\tilde{S}_{\text{max}}} \tilde{l}_w \tilde{\rho} d\tilde{S} \quad (2)$$

with $\tilde{l}_w \equiv l_w/l$ and

$$\tilde{S}_{\text{min}} \equiv \frac{S_{\text{min}}}{S_l} \sim \left(\frac{l_{\text{min}}}{l} \right)^2, \quad \tilde{S}_{\text{max}} \equiv \frac{S_{\text{max}}}{S_l} \sim \left(\frac{L}{l} \right)^{\frac{91}{48}}. \quad (3)$$

The lower cutoff S_{min} is determined by the microscopic length of the system, that is, by the thickness l_{min} of the domain wall: $S_{\text{min}} = \pi l_{\text{min}}^2$. The upper cutoff comes from percolation theory. The area S_{max} of the largest domain is connected with the percolation probability $P(p)$ at percolation threshold $p = p_c$, $P(p = p_c) = \lim_{L \rightarrow \infty} S_{\text{max}}(p_c, L)/L^2 \propto L^{-\beta/\nu}$, with the exponents $\beta = 5/36$ and $\nu = 4/3$ for two-dimensional percolation [6]. Here, $p_c = 0.5$ is assumed for the coarsening dynamics of conventional systems [7–10], which has been numerically confirmed for segregating binary BECs [19].

We assume an asymptotic form for $\tilde{S}_{\text{min}} \ll 1$ and $\tilde{S}_{\text{max}} \gg 1$ (see Fig. 1),

$$\tilde{\rho} \sim \tilde{S}^{-\tau}, \quad \tilde{l}_w \sim \tilde{S}^{D/2} \quad (4)$$

with different exponents τ and D in the two dynamic-scaling regimes: the microscopic regime ($\tilde{S}_{\text{min}} \ll \tilde{S} \ll 1$) with $(\tau, D) = (\tau_{\text{mic}}, D_{\text{mic}})$ and the macroscopic regime ($1 \ll \tilde{S} \ll \tilde{S}_{\text{max}}$) with $(\tau, D) = (\tau_{\text{mac}}, D_{\text{mac}})$. The exponent $1 \leq D \leq 2$ characterizes the fractal behavior of the domain walls. The trivial exponent, $D = 1$, is realized for circular domains with $\tilde{l}_w \sim 2\sqrt{\pi}\tilde{S}$, for example. The upper bound, $D = 2$, comes from the spatial dimension of our system. Under our assumption, the length normalization condition (2) is reduced to

$$\lambda_{\text{mic}} + \lambda_{\text{mac}} + \lambda_P \sim 1 \quad (5)$$

with $\lambda_{\text{mic}} = \int_{\tilde{S}_{\text{min}}}^1 \tilde{l}_w \tilde{\rho} d\tilde{S}$ and $\lambda_{\text{mac}} = \int_1^{\tilde{S}_{\text{max}}} \tilde{l}_w \tilde{\rho} d\tilde{S}$. Here, the contribution from the domain walls surrounding the largest (percolation) domain is extracted explicitly as $\lambda_P = l_w(S_{\text{max}})/R$ in Eq. (5).

A theoretical restriction is obtained for τ_{mic} and τ_{mac} by evaluating the reduced condition (5) in the limit of $\tilde{S}_{\text{min}} \rightarrow 0$ and $\tilde{S}_{\text{max}} \rightarrow \infty$. All terms on the left-hand side of Eq. (5) must be on the order of unity or zero. From Eqs. (3) and (4), we obtain $\tilde{l}_w(S_{\text{max}}) \sim \tilde{S}_{\text{max}}^{D_{\text{mac}}/2} \lesssim \tilde{S}_{\text{max}} \sim (L/l)^{91/48}$; thus, $\lambda_P \rightarrow 0$ in the limit. Substituting Eq. (4) into λ_{mic} , we have the restriction for τ_{mic} ,

$$\tau_{\text{mic}} \leq D_{\text{mic}}/2 + 1. \quad (6)$$

Similarly, the restriction for τ_{mac} is derived as $\tau_{\text{mac}} \geq D_{\text{mac}}/2 + 1$. A similar evaluation is available for the normalization condition of area, $1/2 = \int S \rho dS$ [23]. This condition yields $\tau_{\text{mic}} \leq 2$ and

$$\tau_{\text{mac}} \geq 2. \quad (7)$$

The restriction (7) has been obtained in Ref. [7] and is consistent with the theory of phase ordering percolation: $\tau_{\text{mac}} = \tau_F = 187/91 > 2$. Since we are primarily interested in the microscopic regime, we do not discuss the macroscopic regime further.

To validate our theory, the obtained restriction (6) in the microscopic regime is compared with results in conventional coarsening systems of nonconserved and conserved order parameters at zero temperature [7, 22]. For

nonconserved coarsening in the two-dimensional Ising model (2dIM) [7], the distribution function ρ becomes flat for $S \rightarrow 0$ with $\tau_{\text{mic}} = 0$. On the other hand, 2dIM simulations for the conserved case [22] indicate $\tau_{\text{mic}} = -0.5$. Smaller domains are close to or indeed circular with $D_{\text{mic}} = 1.1$ and $D_{\text{mic}} = 1$ for the former and latter cases, respectively [22]; the restriction (6) is safely satisfied. These results suggest the scaling of exponents in the microscopic regime varies for different types of coarsening systems depending on the “microscopic” dynamics of the domain wall while the exponents in the macroscopic regime take the universal values $\tau_{\text{mac}} = \tau_F$ and $D_{\text{mac}} = 2$ for conventional systems [7, 22].

Our main goal is to study the realizability of the microscopic regime in superfluid systems and identify the scaling behavior. Fortunately, the numerical experiment of segregating binary BECs [19, 20] can be used to achieve this goal.

The segregation dynamics were simulated by solving the coupled Gross-Pitaevskii (GP) equations [24] derived from the GP Lagrangian in a quasi-two-dimensional system, $\mathcal{L} = \int d^2x (\mathcal{P}_\uparrow + \mathcal{P}_\downarrow - g_{\uparrow\downarrow} n_\uparrow n_\downarrow)$. Here, we used $\mathcal{P}_j(\Psi_j) = -\hbar n_j \partial_t \theta_j - \frac{\hbar^2}{2m} (\nabla \sqrt{n_j})^2 - \frac{m}{2} n_j v_j^2 - \frac{1}{2} g_{jj} n_j^2$ ($j = \uparrow, \downarrow$) with atomic mass m , complex order parameter $\Psi_j = \sqrt{n_j} e^{i\theta_j}$, and superfluid velocity $\mathbf{v}_j = \frac{\hbar}{m} \nabla \theta_j$ of the j -component. The Z_2 symmetry breaking is caused by the dynamic instability starting from the initial state with a constant density $|\Psi_\uparrow|^2 = |\Psi_\downarrow|^2 = n/2 = \text{const.}$ and zero velocity $\mathbf{v}_j = 0$ for the coupling constants $g_{\uparrow\downarrow} > g_{\uparrow\uparrow} = g_{\downarrow\downarrow} = g > 0$. For strong segregation in our simulation with $g_{\uparrow\downarrow}/g = 2$ [25], the domain-wall thickness is on the order of the healing length $\xi \equiv \hbar/\sqrt{gmn}$, and we set $l_{\text{min}} = \xi$.

The dynamic instability, triggered by adding small random seeds to the initial state, develops into complicated domain patterns with characteristic length l . We investigated the domain-area distribution after the time when domain patterns emerge clearly at the initial stage of the SSB development. The characteristic length l_0 of the initial domain patterns at that time is of order ξ as determined from the Bogoliubov theory [19]. The size Δx of the numerical grid must be smaller than ξ to precisely simulate the dynamics of the instability and the quantized vortices, which will appear at a later stage. We require a computational system whose size is much larger to satisfy the conditions $\tilde{S}_{\text{min}} \ll 1$ and $\tilde{S}_{\text{max}} \gg 1$ for attaining the coexistence of the microscopic and macroscopic regimes. In our simulation, which utilized the Crank-Nicolson method with periodic boundary conditions, the size and number of the numerical grids are set to meet these requirements during the time development as $\Delta x/\xi = 0.4$ and $L/\Delta x = 4096$, respectively. The numerical results are obtained by averaging 64 samples of the time evolution [26].

Figure 2 shows rescaled plots of the time evolution of

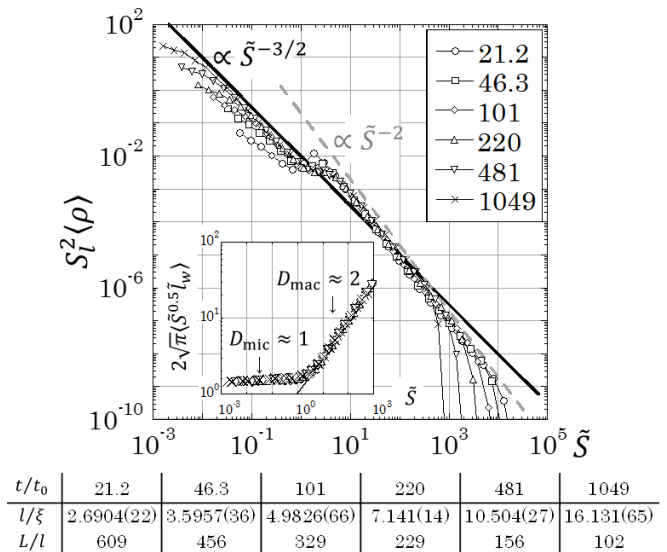


FIG. 2: Dynamic-scaling plot of the averaged domain-area distribution $\langle \rho(S) \rangle$. The graph legends represent the time from $t/t_0 = 21.1$ to 1049 with $t_0 = \frac{\hbar}{gn}$. The table provides information on the corresponding values of l/ξ and L/l . The inset shows the corresponding scaling plot for \tilde{l}_w from $t/t_0 = 101$ to 1049 with the solid line $\tilde{l}_w = \tilde{S}^{1/2}$.

the domain-area distribution (1). Because we have $l \sim l_0$ ($\sim \xi$) in the initial domain patterns, there are fewer domains for $\tilde{S} \lesssim 1$. Thus, the microscopic regime is ill-established, and the rescaled plots of different times do not coincide for $\tilde{S} \lesssim 1$. However, the macroscopic regime is well-established in the early stages ($l/\xi \sim 1$). In late stages ($l/\xi \gtrsim 10$), the microscopic regime becomes well-established by exhibiting the asymptotic form of Fig. 1 with distinct exponents $\tau_{\text{mic}} \approx 3/2$ and $\tau_{\text{mac}} \approx \tau_F$. The dynamic-scaling plot for \tilde{l}_w is also successful in the late stages (see Fig. 2 inset). We have exponents $D_{\text{mic}} \approx 1$ and $D_{\text{mac}} \approx 2$, which is similar to the results of conventional systems [22].

The exponent $\tau_{\text{mic}} = 3/2$ takes the upper limit of the restriction (6) with $D_{\text{mic}} = 1$; that is, the number of domains in the microscopic regime is maximized in the superfluid system. This anomalous behavior is in contrast to the behavior of the above-mentioned conventional systems, while the macroscopic regime shows the universal behavior of percolation criticality.

Generally speaking, the dynamic scaling behavior in the microscopic regime of the domain-area distribution reflects the “microscopic” dynamics of domain walls in the system under consideration. An effective theory that describes the dynamics in our system is quantum hydrodynamics for multi-component superfluids. It is interesting how the hydrodynamics are connected to the anomalous behavior of the microscopic regime in the quantum fluids. The anomaly is suggested to occur in the presence of circular vortex sheets described by the quantum

hydrodynamics.

To provide quantitative evidence of the above suggestion, we introduce a hydrodynamic theory for a vortex sheet between two domains in a multi-component superfluid. Consider a core-less vortex of circulation κn_v with the circulation quantum κ in the superfluid. The vortex core is occupied by a \downarrow -domain surrounded by a sufficiently large \uparrow -domain, where a vortex sheet is formed by the relative velocity between the domains. According to a theory of quantum fluid dynamics [27], such a state is stabilized by centrifugal force due to the rotational superflow by the formation of a circular vortex sheet; however, a flat vortex sheet is dynamically unstable without external forces owing to Kelvin–Helmholtz instability [28]. The radius R of the circular wall is estimated by the equation of pressure equilibrium

$$\Delta p^h = p_{\downarrow}^h - p_{\uparrow}^h = \frac{\sigma_{\text{wall}}}{R} \quad (8)$$

with the tension coefficient σ_{wall} of the wall and the hydrostatic pressure p_j^h ($j = \uparrow, \downarrow$) along the wall in the j -domain.

The theory gives the dynamic-scaling law between n_v and R in the microscopic regime. The pressure \bar{p}_j^h in the bulk, where the superfluid is at rest, is the sum of p_j^h and the hydrodynamic pressure $p_j^d = \frac{1}{2}\rho_j v_j^2$; $\bar{p}_j^h = p_j^h + p_j^d$. Here, ρ_j and v_j are the mass density and superfluid velocity along the domain wall in the j -domain, respectively. The Z_2 symmetry of the Hamiltonian for the multi-component superfluid corresponds to $\bar{p}_{\uparrow}^h = \bar{p}_{\downarrow}^h$. Using this result and substituting $v_{\downarrow}^2 = 0$ and $v_{\uparrow}^2 = (\kappa n_v / 2\pi R)^2$ into Eq. (8), we obtain the dynamic-scaling relation with $S (= \pi R^2)$:

$$|\tilde{n}_v(\tilde{S})| \equiv \sqrt{\frac{l_{\min}}{l}} |n_v(S)| \sim \tilde{S}^{1/4} \quad (\tilde{S}_{\min} \ll \tilde{S} \ll 1). \quad (9)$$

Here, the value of $\xi_{\text{wall}} \equiv (\kappa/2\pi)^2 \rho_{\uparrow} / \sigma_{\text{wall}}$ characterizes a length scale related to the domain wall, and thus, it should be on the order of the wall thickness l_{\min} . In the above discussion, we neglected the existence of domains embedded inside the circular domain wall. However, it is straightforward to show the contribution from such domains is negligible [29].

A circular domain ($D_{\text{mic}} = 1$) shrinks and finally collapses in bulk owing to dissipation or evaporation in the conventional coarsening systems. In contrast, a circular domain with vortices ($n_v \neq 0$) is stable in the presence of rotational superflow. This is why the number of domains in the microscopic regime is statistically enhanced with the upper limit $\tau_{\text{mic}} = 3/2$ in the superfluid system.

The existence of the scaling law (9) is a direct evidence that the microscopic regime is described by the quantum hydrodynamics. We examined this law for the numerical experiments of binary BECs. Figure 3 shows the dynamic scaling plot of the ensemble average of $|n_v(S)|$ for

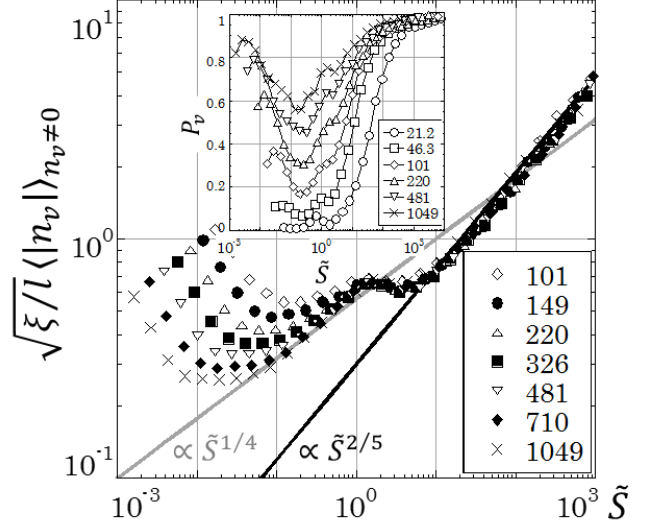


FIG. 3: Dynamic scaling plot of the circulation quantum number $|n_v(S)|$ from $t/t_0 = 101$ to 1049. The inset shows the ratio $P_v(\tilde{S})$ of the number of domains with $|n_v| \neq 0$ from $t/t_0 = 21.2$ to 1049.

domains with $n_v(S) \neq 0$ obtained numerically by integrating the superfluid velocity along domain walls. The scaling law (9) is observed after the power-law behavior is well-established in the microscopic regime. The upturn of the plot at $\tilde{S} \sim \tilde{S}_{\min}$ reflects the core structure of a vortex whose radius is on the order l_{\min} , which was neglected in the derivation of Eq. (9) [30].

The above statistical discussion is meaningful when domains with vortices are dominant over vortex-free domains ($n_v = 0$) in the microscopic regime. The inset of Fig. 3 shows the plot of the probability $P_v(\tilde{S})$ in which a domain of \tilde{S} has $n_v \neq 0$. The probability is negligible in the initial domain pattern because the Bogoliubov modes excited by dynamic instability do not cause momentum exchange, leading to a relative flow between two superfluids [31]. However, superflow is locally induced by complex motions of domain walls, and quantized vortices are nucleated owing to the Kelvin–Helmholtz instability. As a result, the probability P_v grows with time, becoming substantially large at a late stage, where almost domains in the microscopic regime contain vortices. This result clearly shows that the influence of superflow is essential to the understanding of later-stage dynamics.

Interestingly, the dynamic-scaling plot for circulation number n_v yields a different scaling behavior with $|\tilde{n}_v| \sim \tilde{S}^{2/5}$ in the macroscopic regime. This suggests that distribution of velocity or vorticity may obey a different scaling law or power law for length scales larger than the average distance l between domain walls. This situation is similar to the hierarchy in quantum turbulence, where the Kolmogorov or semi-classical power law is realized over length scales larger than the average distance be-

tween vortex lines and a different power law is expected for smaller length scales [32]. In this sense, investigation into the aspects of turbulence is fruitful future work of our system.

To reinforce our hypothesis on phase ordering dynamics, the theory can be applied to different multi-component superfluids, where vortex sheets are stabilized. An important application is quasi-two-dimensional $^3\text{He-A}$ confined in a slab system, where chiral domain walls were recently visualized [33]. The order parameter is the vector field \hat{l} that represents the direction of orbital angular momentum of the Cooper pair in the dipole-locked $^3\text{He-A}$. A one-to-one correspondence exists between the \hat{l} field of $^3\text{He-A}$ and the (pseudo)-spin field of spinor (binary) BECs, according to the Mermin–Ho relation that represents vorticity distribution due to the vector-field texture [34–36]. In this sense, these superfluid systems can show a similar effect even if the “microscopic” dynamics differs between the $^3\text{He-A}$ and spinor (binary) BECs. By quenching the $^3\text{He-A}$ system to the superfluid phase from the normal fluid phase, domain growth should occur. Then, the characteristic domain-area distribution, as is illustrated in Fig. 1, will be observed when the characteristic domain size is much larger than thickness of domain wall and much smaller than the system size.

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- [1] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, 1995).
- [2] *Topological Defects and the Non-Equilibrium Dynamics of Symmetry Breaking Phase Transitions*, edited by Y. M. Bunkov and H. Godfrin, Vol. 549 of NATO Advance Science Institute, Series C: Mathematical and Physical Sciences (Kluwer, Dordrecht, 2000).
- [3] A. Onuki, *Phase Transition Dynamics* (Cambridge University Press, Cambridge, 2002).
- [4] T. Vachaspati, *Kinks and Domain Walls: An Introduction to Classical and Quantum Solitons* (Cambridge University Press, Cambridge, 2006).
- [5] A. J. Bray, *Adv. Phys.* **43**, 357 (1994).
- [6] D. Stauffer and A. Aharony, *An Introduction to Percolation*, 2nd ed. (Taylor and Francis, London, 1994).
- [7] J. J. Arenzon, A. J. Bray, L. F. Cugliandolo, and A. Sicilia, *Phys. Rev. Lett.* **98**, 145701 (2007); A. Sicilia, J. J. Arenzon, A. J. Bray, and L. F. Cugliandolo, *Phys. Rev. E* **76**, 061116 (2007).
- [8] A. Sicilia, J. J. Arenzon, I. Dierking, A. J. Bray, L. F. Cugliandolo, J. Martínez-Perdiguero, I. Alonso, and I. C. Pintre, *Phys. Rev. Lett.* **101**, 197801 (2008).
- [9] J. Olejarz, P. L. Krapivsky, and S. Redner, *Phys. Rev. Lett.* **109**, 195702 (2012).
- [10] A. Tartaglia, L. F. Cugliandolo, and M. Picco, *Europhys. Lett.* **1116**, 26001 (2016); L. Cugliandolo, *Phase ordering kinetics, aggregation and percolation in two dimensions*, Plenary lecture in STATPHYS26, Lyon, France (2016).
- [11] K. Damle, S. Majumdar, and S. Sachdev, *Phys. Rev. A* **54**, 5037 (1996).
- [12] S. Mukerjee, C. Xu, and J. E. Moore, *Phys. Rev. B* **76**, 104519 (2007).
- [13] H. Takeuchi, K. Kasamatsu, M. Tsubota, and M. Nitta, *Phys. Rev. Lett.* **109**, 245301 (2012).
- [14] K. Kudo and Y. Kawaguchi, *Phys. Rev. A* **88**, 013630 (2013).
- [15] M. Karl, B. Nowak, and T. Gasenzer, *Phys. Rev. A* **88**, 063615 (2013).
- [16] K. Kudo and Y. Kawaguchi, *Phys. Rev. A* **91**, 053609 (2015).
- [17] J. Hofmann, S. S. Natu, and S. Das Sarma, *Phys. Rev. Lett.* **113**, 095702 (2014).
- [18] L. A. Williamson and P. B. Blakie, *Phys. Rev. Lett.* **116**, 025301 (2016); *Phys. Rev. A* **94**, 023608 (2016).
- [19] H. Takeuchi, Y. Mizuno, and K. Dehara, *Phys. Rev. A* **92**, 043608 (2015).
- [20] H. Takeuchi, *J. Low Temp. Phys.* **183**, 169 (2016).
- [21] A. Bourges and P. B. Blakie, *Phys. Rev. A* **95**, 023616 (2017).
- [22] A. Sicilia, Y. Sarrazin, J. J. Arenzon, A. J. Bray, and L. F. Cugliandolo, *Phys. Rev. E* **80**, 031121 (2009).
- [23] The integral represents the total area occupied by \uparrow - or \downarrow -domains, divided by the system area L^2 , yielding $\int SpdS = 1/2$ for the case of the Z_2 symmetry breaking.
- [24] C. J. Pethick and H. Smith, *Bose–Einstein Condensation in Dilute Gases*, 2nd ed. (Cambridge University Press, Cambridge, 2008).
- [25] The coupling constants are controllable by changing the atomic interaction through the Feshbach resonance and the oblateness of the trapped condensates. Strong segregation can be achieved, for example, S. B. Papp, J. M. Pino, and C. E. Wieman, *Phys. Rev. Lett.* **101**, 040402 (2008). Since spinor BECs can be described effectively by the same hydrodynamics, the system is also available for examining the scaling behavior. See also the references in *e.g.*, Ref. [18].
- [26] The ensemble averages, except for the average of l , were taken by considering the periodic boundary conditions; averaged quantities are calculated by averaging 64 samples after the secondary average over the 8×8 pseudo-samples obtained by shifting the field data of a single simulation in the x and y directions by $i \times L/8$ and $j \times L/8$ with integers i and j ($0 \leq i, j \leq 7$), respectively. The length $L/8$ is much larger than the characteristic length l of the field, so the secondary average improves the statistical analysis substantially.
- [27] S. Hayashi, M. Tsubota, and H. Takeuchi, *Phys. Rev. A*, **87**, 063628 (2013).
- [28] H. Takeuchi, N. Suzuki, K. Kasamatsu, H. Saito, and M. Tsubota, *Phys. Rev. B* **81**, 094517 (2010).
- [29] The contribution of embedded domains to the circulation number n_v is statistically negligible. The contributions to the circulation from the considered domain and the domains embedded within it are calculated analytically and compared statistically according to the dynamic scaling

($\tilde{S}_{\text{min}} \ll \tilde{S}_{\text{out}} \ll 1$) in the context of our dynamic scaling hypothesis, where domain structures should be statistically similar in the regime with a unique scaling behavior.